

Death, Taxes and Inequality

Complexity Emerges from a Simple ABM of an Economy
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CEIICH-CEPHCIS-UNAM

John C. Stevenson
jcs@alumni.caltech.edu

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Long Beach

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Agenda

- The greatest engineering problem of our generation
- “A New Kind Of Science”
- Complex Adaptive Systems
- Modern Coexistence Theory
- Minimal model of system
- Methods
- Results
- Future work

The Engineering Problem

The pasture is the commons. All may graze; all are tempted to push their share to the utmost, in short time the whole stock perishes.

William Llyod, Two Lectures on the Checks to Population (Lloyd, 1833)

- At first glance it appears our social systems are grossly inefficient
 - ▶ education
 - ▶ health care
 - ▶ tax structures
 - ▶ economic inequality
 - ▶ global warming
- Quantification of these inefficiencies gives a surprising result
 - ▶ these systems are highly optimized
 - ▶ they are optimized to preserve the status quo of inequality and privilege. (Zinn, 2015; Giridharadas, 2019)
- The problem is as much sociology and political science as it is engineering

“A New Kind of Science”

- Computational science joins the theoretical and experimental branches
- Rigorous tools bring new understandings of complexity
- Cellular automata (Wolfram and Gad-el Hak, 2003)
- Generative social science (Schelling, 1969; Epstein, 1999)
 - ▶ Agent Based Modeling
- Evolutionary Optimization (EO)
 - ▶ Genetic algorithms (Holland, 1992)
 - ▶ Genetic programming
 - ★ rediscovery of patents for electric circuits (Koza, 1992)
 - ★ Inverse agent-based modeling (Stevenson, 2023c)
 - ★ Inverse generative social science (Epstein, 2023; Vu et al., 2019)
- not AI /large language models

The social insects ... represent the most advanced forms of social evolution ... are based on rigid genetic programming rather than cultural evolution.
E. O. Wilson. Sociobiology (Wilson, 2000)

Complex Adaptive Systems

- The emergence of complexity extends across all scientific fields
 - ▶ sensitive to initial conditions
 - ▶ multiple basins of attraction
 - ▶ frequently out of equilibrium
- Complex Adaptive Systems (CAS) are complex systems under evolutionary selection pressure
- Two types or levels of adaption (Wilson and Kirman, 2016)
 - ▶ Selection of individuals employing adaptive strategies (CAS2)
 - ★ **within-group selection**
 - ★ individual “struggle for survival” (Gause, 1934)
 - ★ “The Invisible Hand” (Smith, 1937)
 - ★ “tragedy of the commons” (Ostrom, 1990)
 - ★ beavers effects on ecosystem (multispecies)
 - ▶ Selection at the system level (CAS1)
 - ★ **between-group selection**
 - ★ social insect communities (eusocial)
 - ★ Water Striders (*Aquarius remigis*) (single species)

Modern Coexistence Theory

- Coexistence or exclusion is a measure of relative fitness of groups (Chesson, 2000, 2018)
- Permits CAS2 and CAS1 determinations
- Based on the Lotka-Volter model (Lotka, 1932; Volterra, 1926; Chesson, 2018; Kot, 2001)

$$\frac{dN_i(t)}{dt} = r_i N_i \left(1 - \sum_{j=1}^2 \alpha_{ij} N_j\right), i = 1, 2 \quad (1)$$

where N_i is the population density of species i and j r_i and $1/\alpha_{ij}$ are the intrinsic growth rate and carry capacity α_{ij} , $i \neq j$ is a competitive (commensal) coefficient of i on j

- reduces to the Verhulst's continuous and discrete logistic growth equations for a single species. (Verhulst, 1838; Murray, 2002).
- The mutually invasive criteria:

$$\alpha_{ij} > \alpha_{jj} \quad (2)$$

must hold that for coexistence both species as invaders are capable of showing positive growth when invading a resident population. (Chesson, 2018).

Minimal Model of a System

Roughgarden's classification of models (Roughgarden et al., 1996)

- Minimal model of an idea
- Synthetic model of a system
 - ▶ **Matches** macroscopic results of model to **empirical data**
 - ▶ Rules are hand crafted or automatically generated (iGSS)
 - ▶ **Iterated calibration external to model (exogenous)**
 - ▶ Examples
 - ★ Epidemiological models
 - ★ Economics models
 - ★ iGSS: **bird flocking** (Greig R. and Arranz J.,2021)
 - ★ iGSS: artificial anasazi (Gunaratne C. and Garibay I.,2017)

Minimum Model of a System (cont.)

It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.

Albert Einstein (Einstein, 2011)

Minimal model of a system

- **Freely evolves**, does not attempt to calibrate to empirical data
- May evolve outside of the model (**exogenous**)
 - ▶ Iterated Prisoner Dilemma (Axelrod and Hamilton, 1981; Fogel, 1993; Lindgren, 1991; Lindgren and Nordahl, 1994; Miller, 1996; Skyrms, 2017, 2004)
- May evolve internally as part of the model (**endogenous**)
 - ▶ Evolution of genetic characteristics of individuals
 - ★ Sugarscape (Epstein J.M. and Axtel R., 1996)
 - ★ Alarm Calling and Feeding Restraint (Pepper and Smuts, 2000)
 - ★ Demographic Prisoner's Dilemma (Epstein J. W., 1998)

Methods

- Minimal Model of a Foraging Economy (Epstein and Axtell, 1996)
 - ▶ single resource, metabolism 75% of max cell resource
 - ▶ asexual (haploid) reproduction with infertility as probabilistic birth
 - ▶ intrinsic growth rate and carry capacity

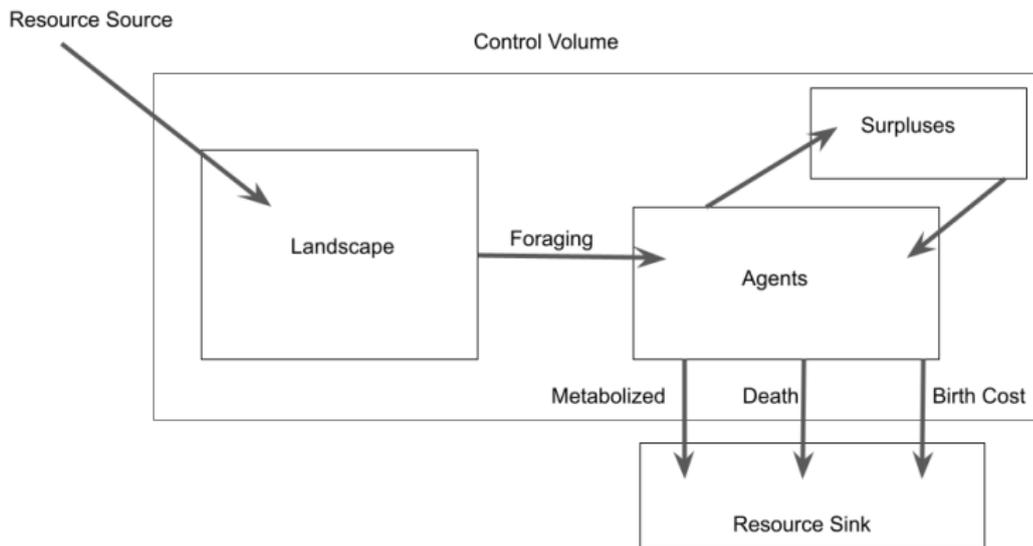


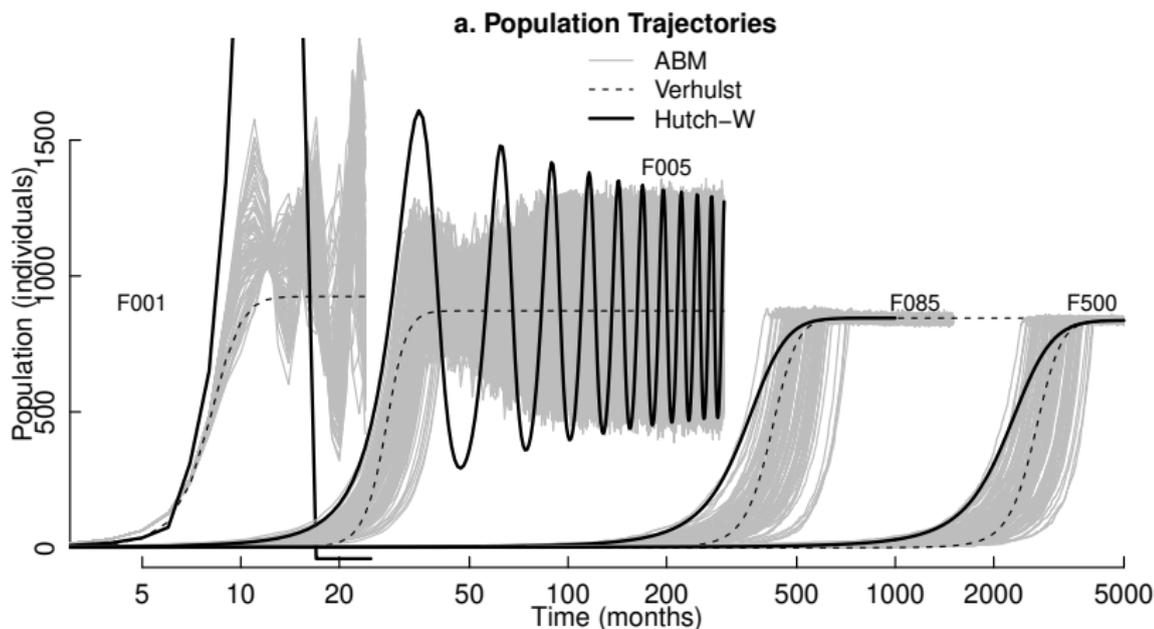
Figure 1. Control Volume of Resource Flows

Minimal Model of a Foraging Economy

- ABM of foraging society (Epstein and Axtell, 1996; Stevenson, 2022)
- **Identical Agents on an equal opportunity (flat) landscape**
- Does not attempt to calibrate or optimize to empirical data
- Endogenous objective function and reproduction Epstein (1999); Stevenson (2023a)
- Simple “struggle for existence” (Gause, 1934)
- **Too simple ? Four comparisons** (Stevenson, 2022)
 - ▶ Single species discrete population dynamics (Kot, 2001; Murray, 2002)
 - ▶ Discrete stochastic gene frequency models (Ewens, 2004; Cannings, 1974; Moran, 1958)
 - ▶ Modern Coexistence Theory (Chesson, 2000)
 - ▶ Income Distributions (Stevenson, 2024)

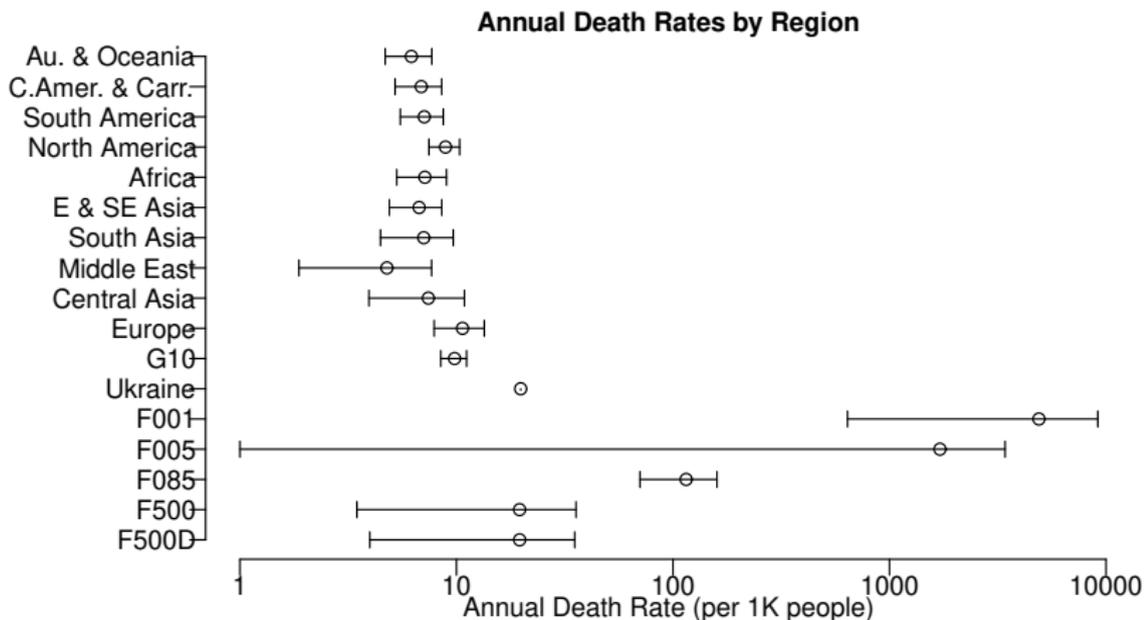
Single Species Logistic Growth Models

- Continuous Verhulst Model $\frac{dN(t)}{dt} = rN(1 - \frac{N}{K})$ (Verhulst, 1838; Kot, 2001) where N is the population size, t is time, r is the intrinsic growth rate, K is the carry capacity, and t is time
- Hutchinson-Wright Time Delayed Discrete Model (Hutchinson, 1961; Wright, 1955)
- growth rate given by $F00x$ where 1 in x is probability of reproduction
- **regimes of population volatility**: steady, oscillatory, and chaotic
- **strong selection pressure** to maximum growth rate (constant environment)



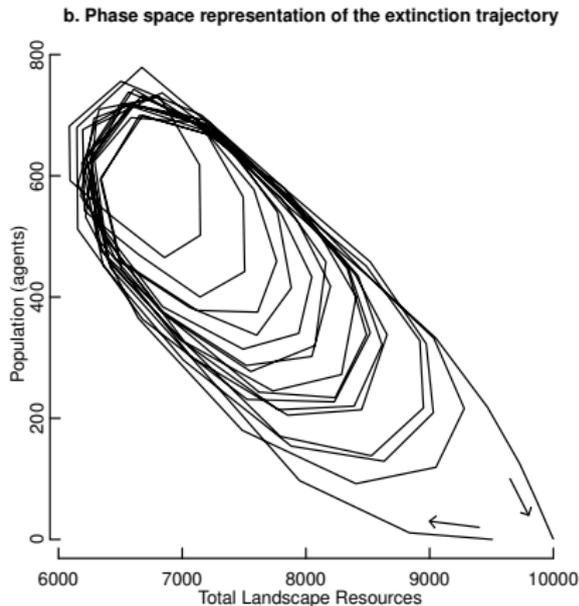
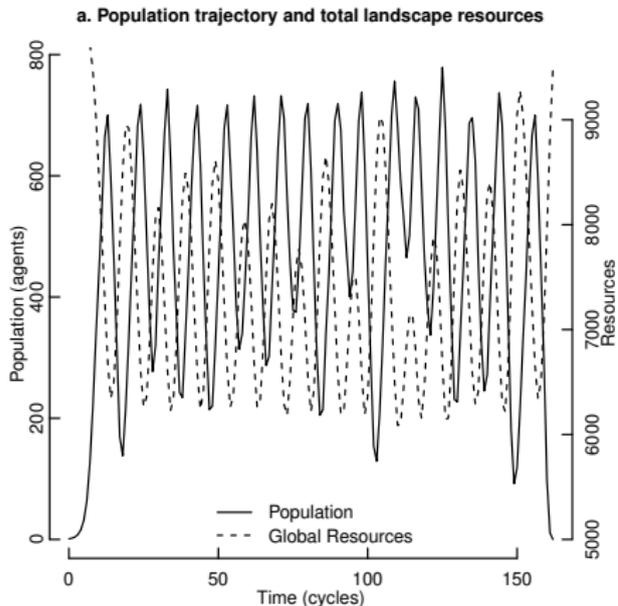
Changes For Comparisons to Real Economies

- Reproduction probability reduced to match empirical death rates (**F500**)
- Death by natural causes (Finite Lifespan)
 - ▶ model initially is only death by "starvation" (**no FL**)
 - ▶ flat probability of death between 400 and 600 cycles (FL)
 - ▶ same death rate, about half the deaths are from natural causes



Chaotic Extinction of a Subsistence Economy

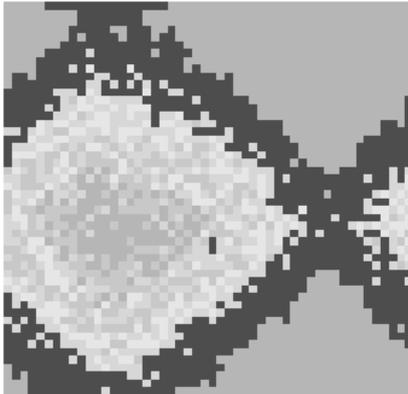
Global Measures of an Extinction of a Subsistence Economy



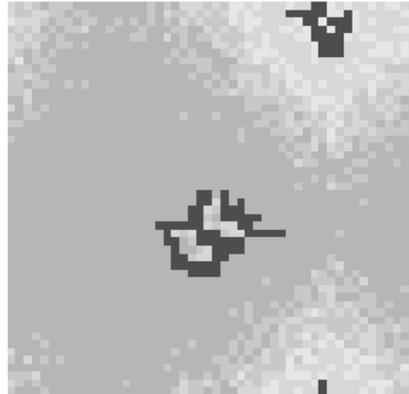
a) The coupled, chaotic magnitude oscillations of levels of population and total landscape resources for a simple, subsistence economy (flp1bc0m4). While the total landscape resources always remain well above the level necessary to keep a good fraction of the population alive, the extinction nonetheless occurs. b) Representation in phase space of the levels of population and total landscape resources about the attractor.

Microstructure of an Extinction Event

Generation 144

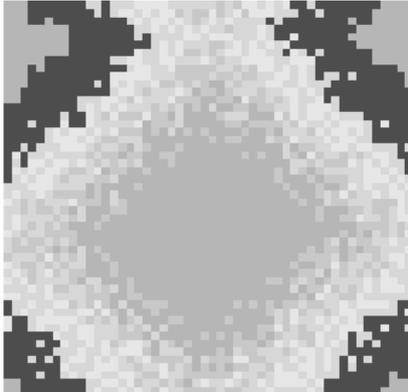


Generation 149

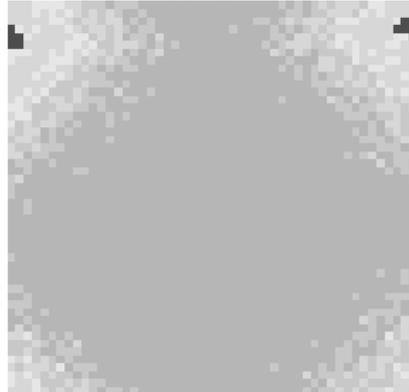


Legend: Cell Resource: 0 (white) to 4 (dark gray), Agent occupying cell (black)

Generation 158



Generation 161



Stochastic Behavior of Genes in Finite Populations

- Wright-Fisher fixation probability π for neutral alleles

$$\pi = i/2N \quad (3)$$

where i is the initial population of one of the two alleles and the total population is $2N$. $2N$ for the haploid population allows comparison with diploid populations of standard Wright-Fisher models. (Fisher, 1923; Wright, 1931)

Probability of Allele Fixation and ABM Death Rates

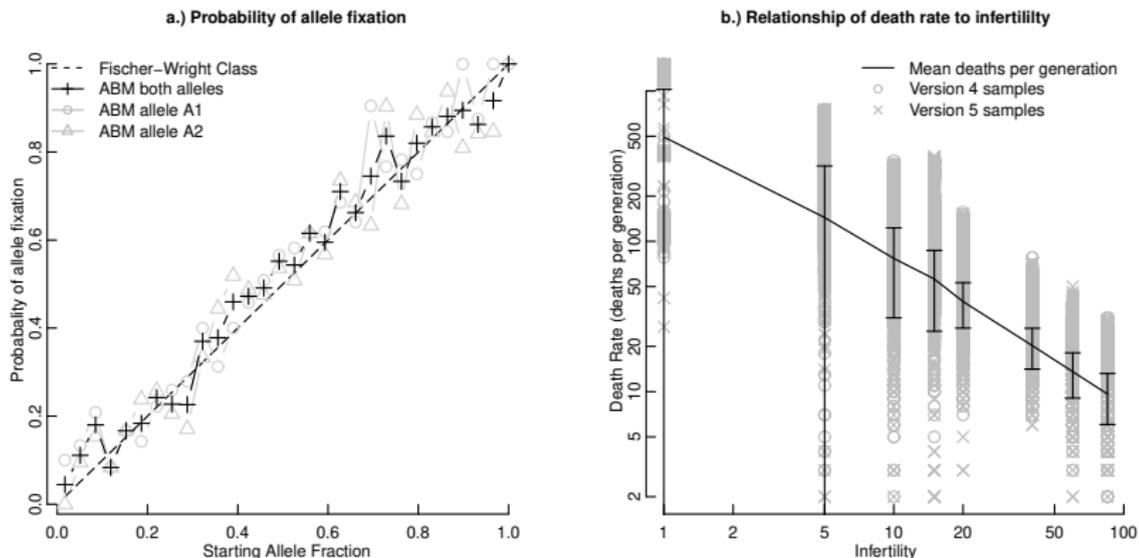


Figure 5. a) Probability of fixation of an allele as a function of it's starting allele ratio (eq. 3) across infertilities of 1, 5, 10, 15, 20, 40, 60 and 85. b) Relationship of the ABM parameter infertility to the generated death rates.

Stochastic Behavior of Genes: Overlapping Generations

- While the probability of fixation was straightforward, the fixation time was more challenging
- Cannings rule for stochastic reproduction with overlap $y(t)_i$; (Cannings, 1974; Moran, 1958)
- The variance, $\sigma(t)^2$, of $y(t)_i$ is shown to be

$$\sigma(t)^2 = \frac{1}{2N-1} [d(t) + 0 + d(t)] = \frac{2d(t)}{2N-1} \quad (4)$$

where $d(t)$ is the number of deaths (and with constant population births). **This variance can be directly computed in the ABM.**

- The mean time to absorption \bar{t}_{abs} is given as (Ewens 2004)

$$\bar{t}_{abs} = -(4N-2)[p \log(p) + (1-p) \log(1-p)] / \sigma(t)^2 \quad (5)$$

where p is the ratio of the number of one allele A_1 to the total population.

Stochastic Behavior of Genes: Absorption Times

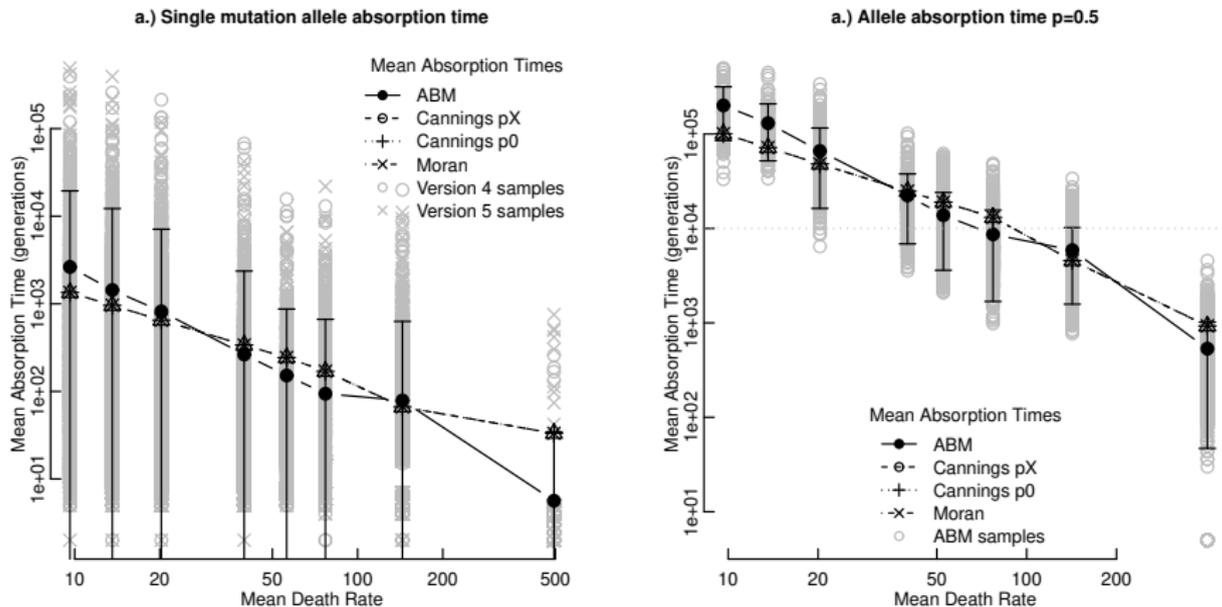


Figure 6. The mean time to fixation for a) a single mutation and b) for $p = 1/2$ with no selection pressure.

Stochastic Behavior of Genes: Weak Selection: Moran

The probability the next individual selected to die is A_1 is $\mu_1 i / \{\mu_1 + \mu_2(2N - i)\}$ where i is the number of A_1 alleles in the $2N$ population and $\frac{\mu_1}{\mu_2}$ defines the selective advantage. The probability of fixation:

$$\pi_i = \{1 - (\mu_1/\mu_2)^i\} / \{1 - (\mu_1/\mu_2)^{2N}\} \quad (6)$$

By defining $\mu_1/\mu_2 = 1 - s/2$ with s small and positive, Equation (8) can be approximated as

$$\pi(x) = \{1 - \exp(-\alpha x/2)\} / \{1 - \exp(-\alpha/2)\} \quad (7)$$

where $x = i/2N$ and $\alpha = 2Ns$.

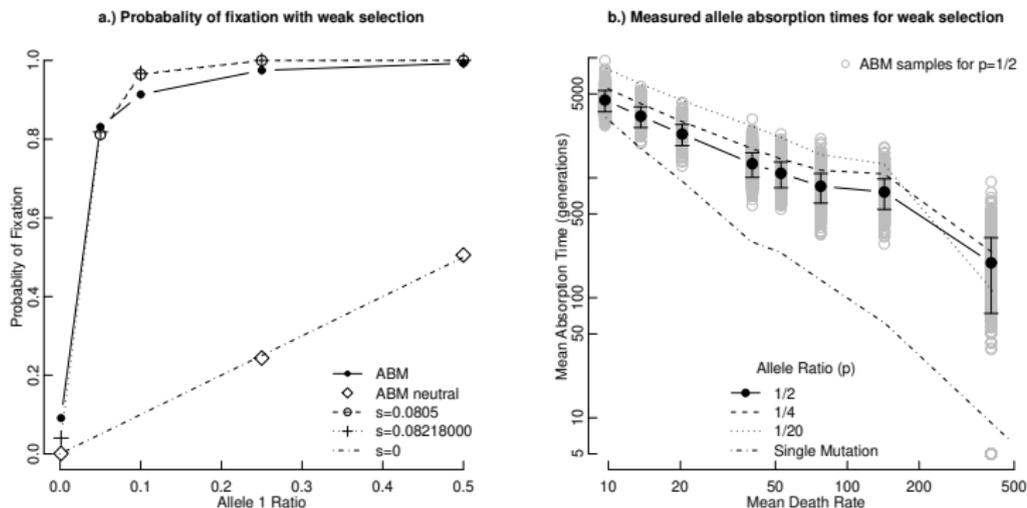


Figure 7. a) The probability of fixation of A_1 . b) The mean time to fixation by mean death rate for sample initial allele ratios.

Chesson's Modern Coexistence Theory (Chesson, 2000)

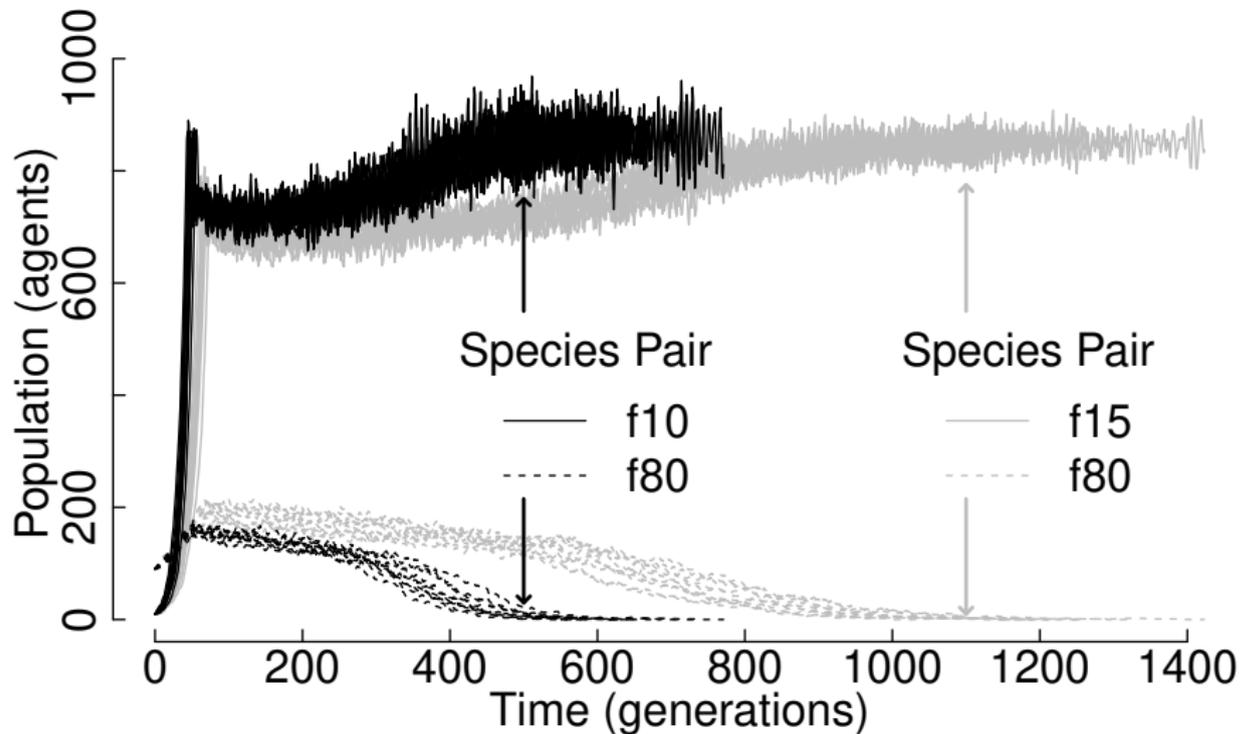


Figure: Two Species Exclusion in a Stable Environment

Chesson's Modern Coexistence Theory

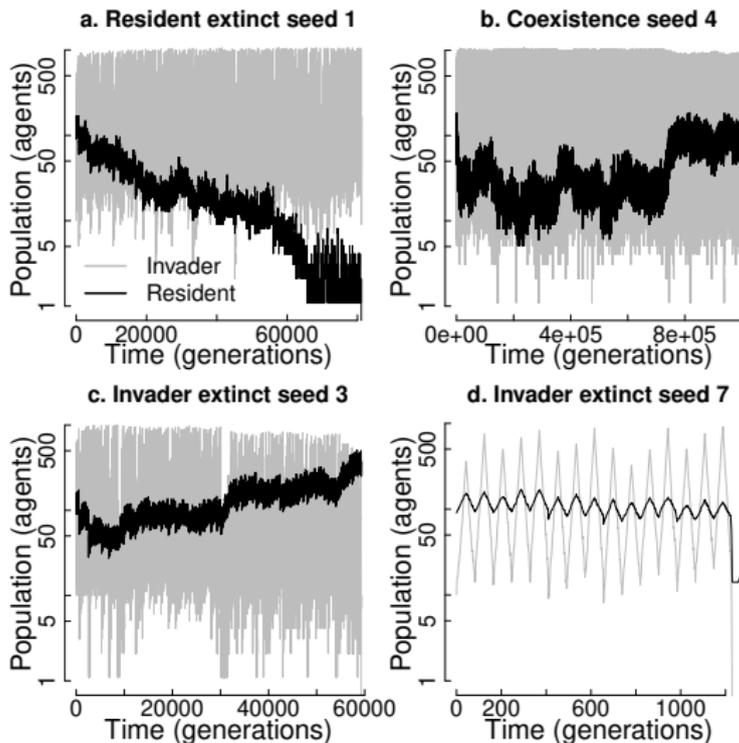


Figure: Trajectories for Seasonal RMLC model ecosystem (Stevenson, 2023b)

Actual Economic Data and Implied Distributions

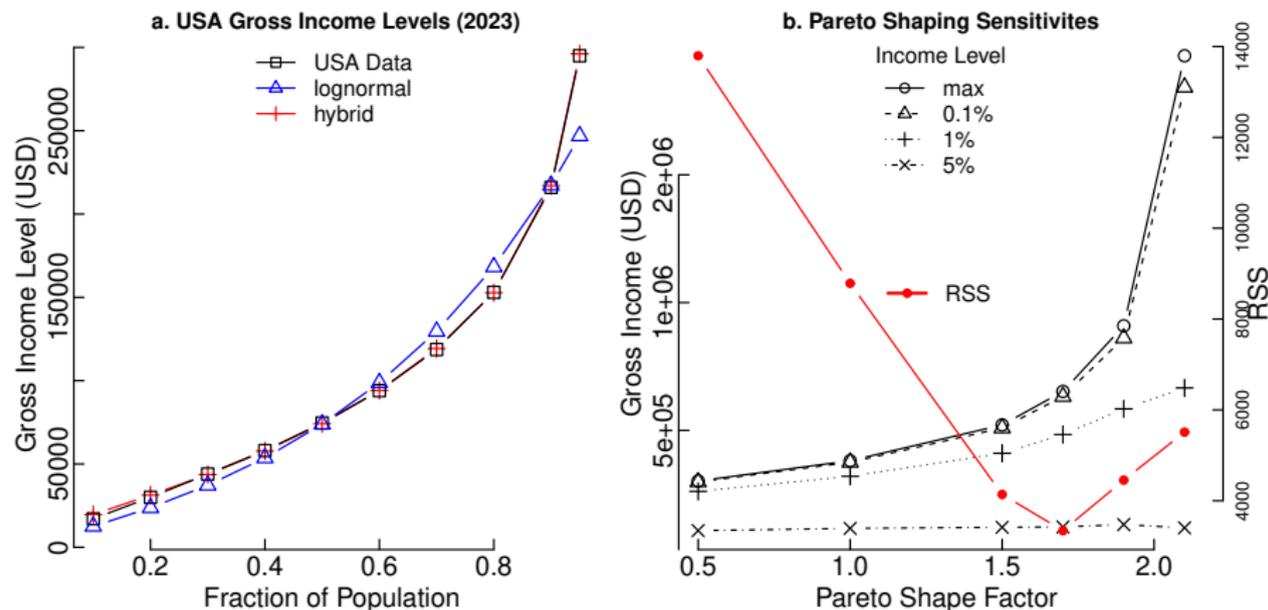
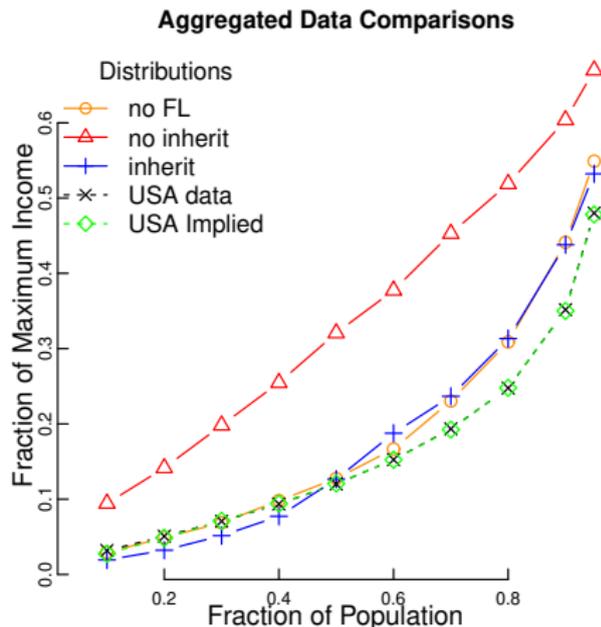
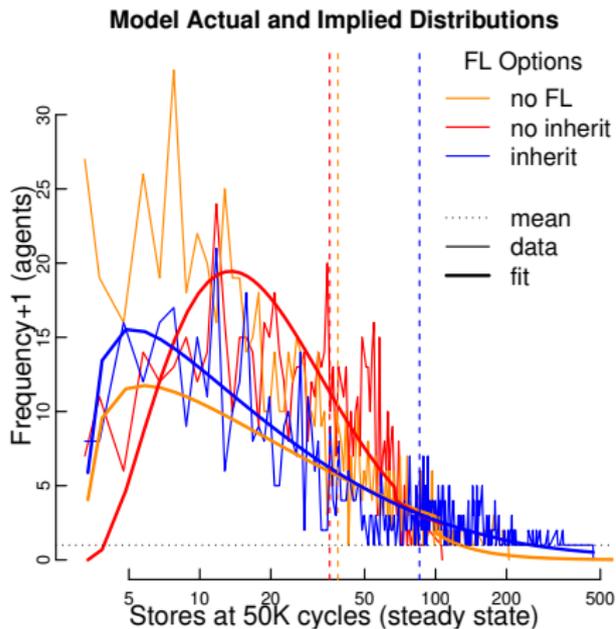


Figure: a) USA data (United States Census Bureau, 2022) and lognormal and hybrid fits. b) Pareto calibration optimization.

Model Actual, Implied, and Aggregated distributions



Results

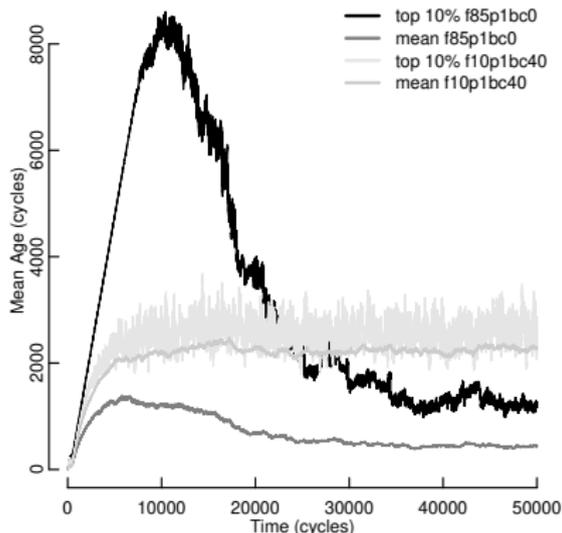
The economy is a complex system. It is constantly forming itself from the bottom up, and it does not operate in a state of equilibrium.
W. Brian Arthur (Arthur, 2009)

- Non-equilibrium
- Coexistence
- Towards Eusociality
- Applications
- Future Work

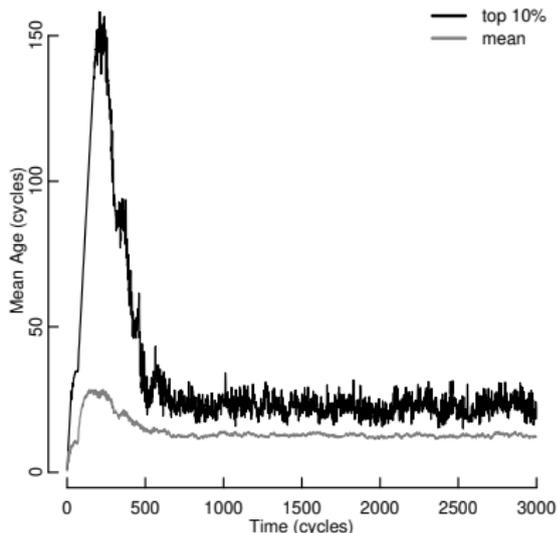
Non-equilibrium Modeling

Mean Age Relaxation Times for Model Surplus Economies

a.) Elite and total mean ages for f85p1bc0m3 and f10p1bc40m3



b.) Elite and total mean Ages for f10p1bc0m3



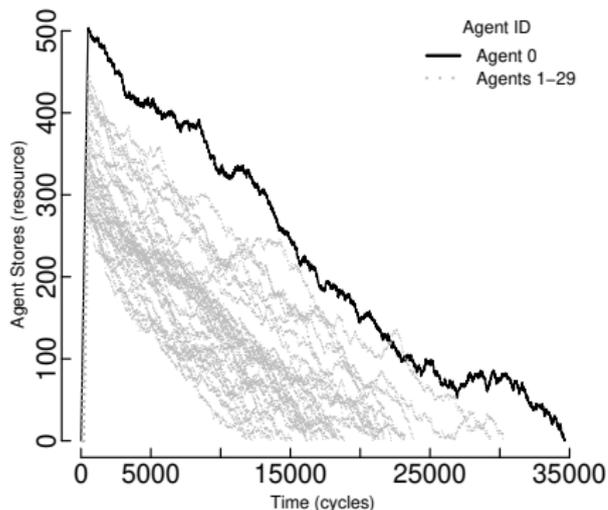
a) Elite (top 10%) and overall mean ages for surplus economies f85p1bc0 and f10p1bc40. b) Elite (top 10%) and overall mean ages for surplus economy f10p1bc0. The large inequalities are generated during the population's growth to carry capacity and persist for periods much longer than the time to reach carry capacity. Model economy f85p1bc0m takes the longest time to attain steady state at over 30,000 cycles.

Non-equilibrium Modeling (cont.)

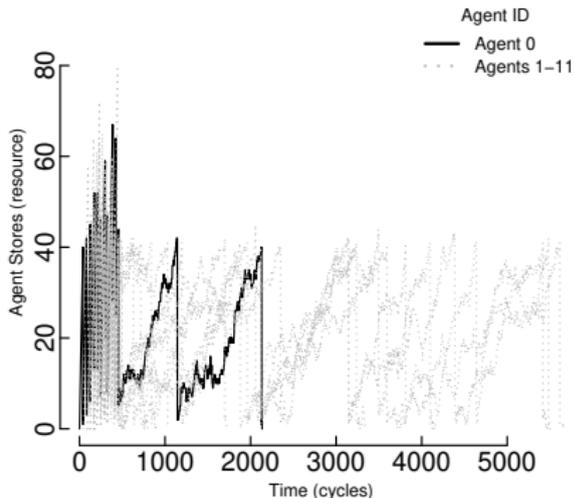
- spatial and temporal histories for individuals available

History of the Founding Agents' Wealth for Low Fertility and Birth Cost Societies

a.) Founding Agents' Wealth for Low Fertility

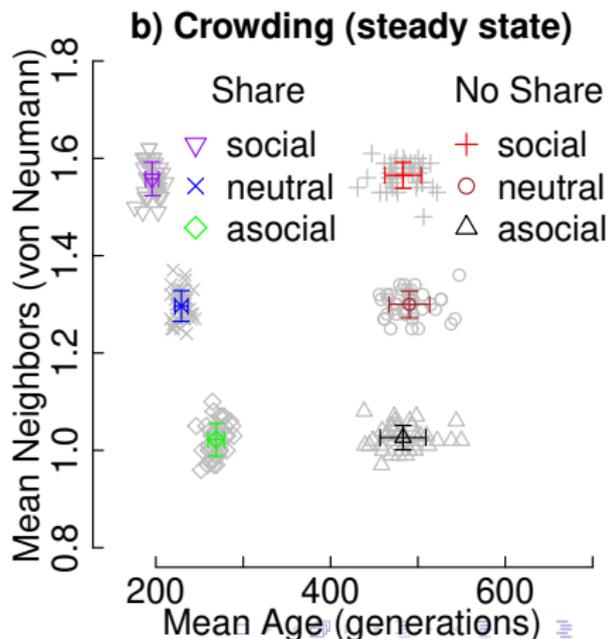
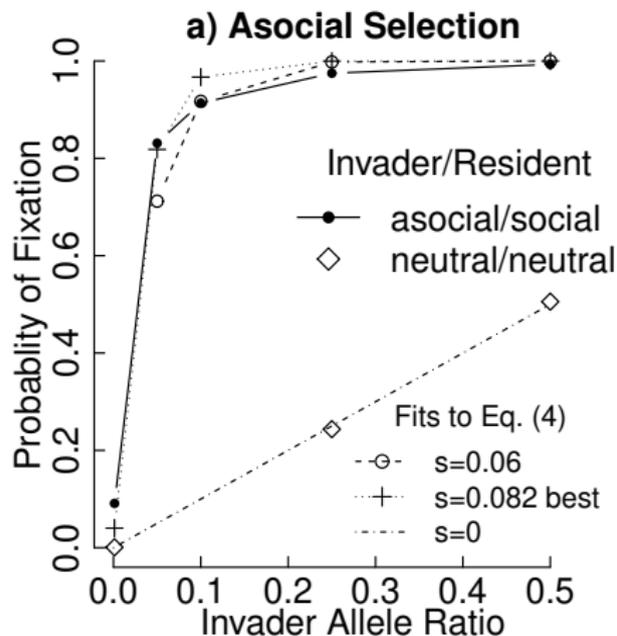


b.) Founding Agents' Wealth History for Birth Cost



Sociality

- Agents move to the highest, closest resource within view, sociality gene(s) breaking ties
 - random tie break, no selection pressure
 - extrovert (social) moves to most crowded location
 - introvert (asocial) moves to least crowded location



Sociality (cont.)

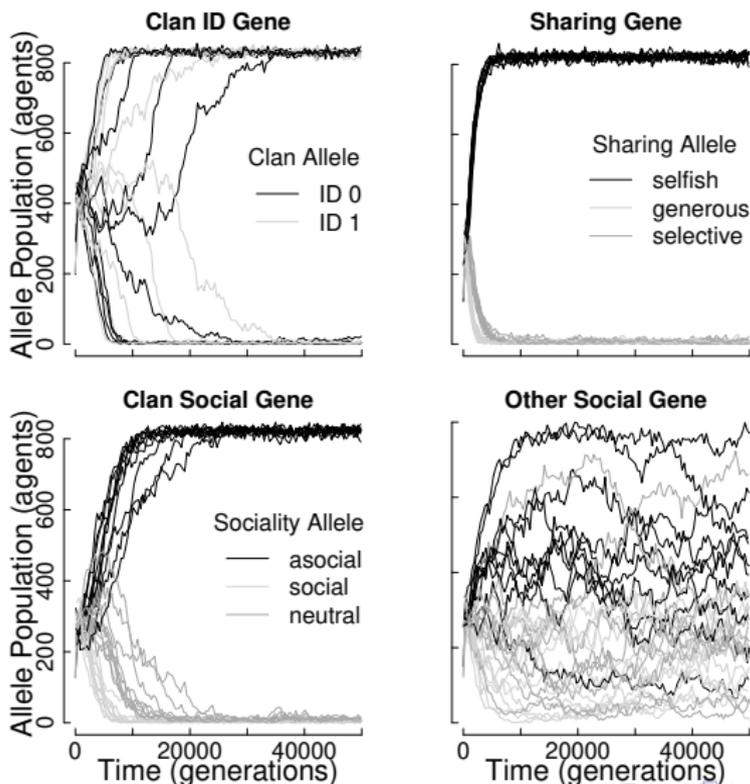
- Divide population into two groups (own and other)

Clan Social Gene	Other Social Gene	Label	Behavior
0	0	loner	avoids everyone
1	0	clannish	seeks clan, avoids others
2	0	shy	avoids others
0	1	aggressive	avoids clan, seeks others
1	1	gregarious	seeks everyone
2	1	friendly	seeks others
0	2	outcast	avoids clan
1	2	homebody	seeks clan
2	2	neutral	no preferences

Table: Sociality alleles for own clan and other clans genes

Evolution of Sociality and Sharing with Clans

- sharing alleles are none, within group, and with all



Economics of Sharing and Sociality

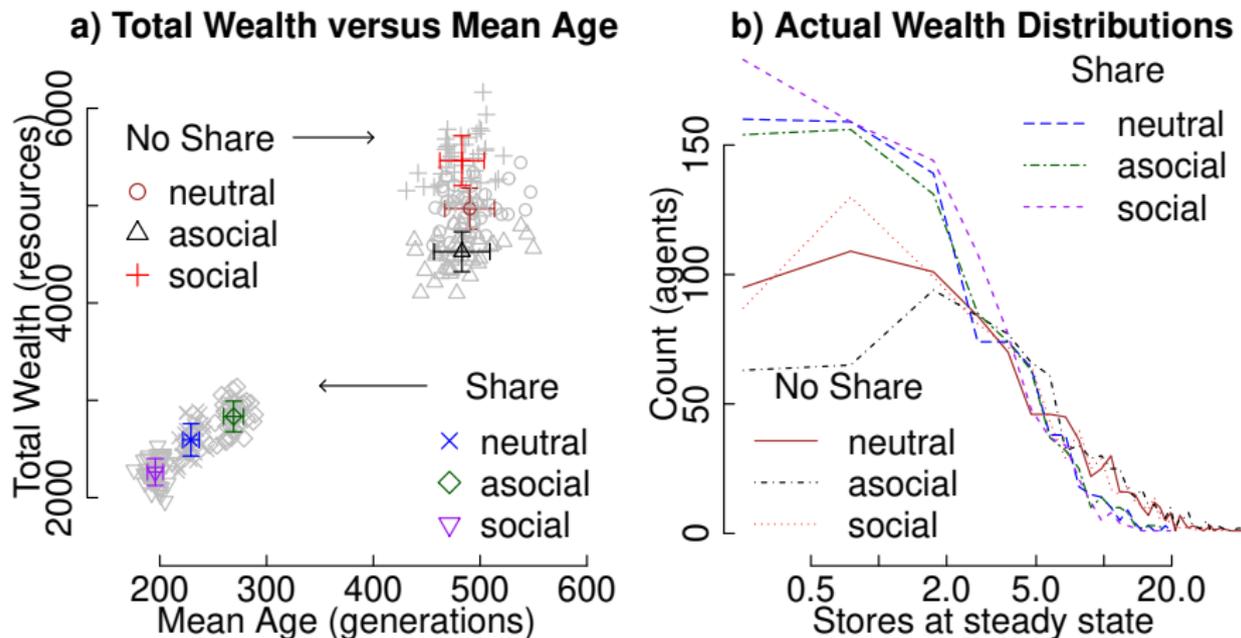


Figure: Single group sharing and sociality effects on total wealth at steady state

Inequality and Multigroup Economics

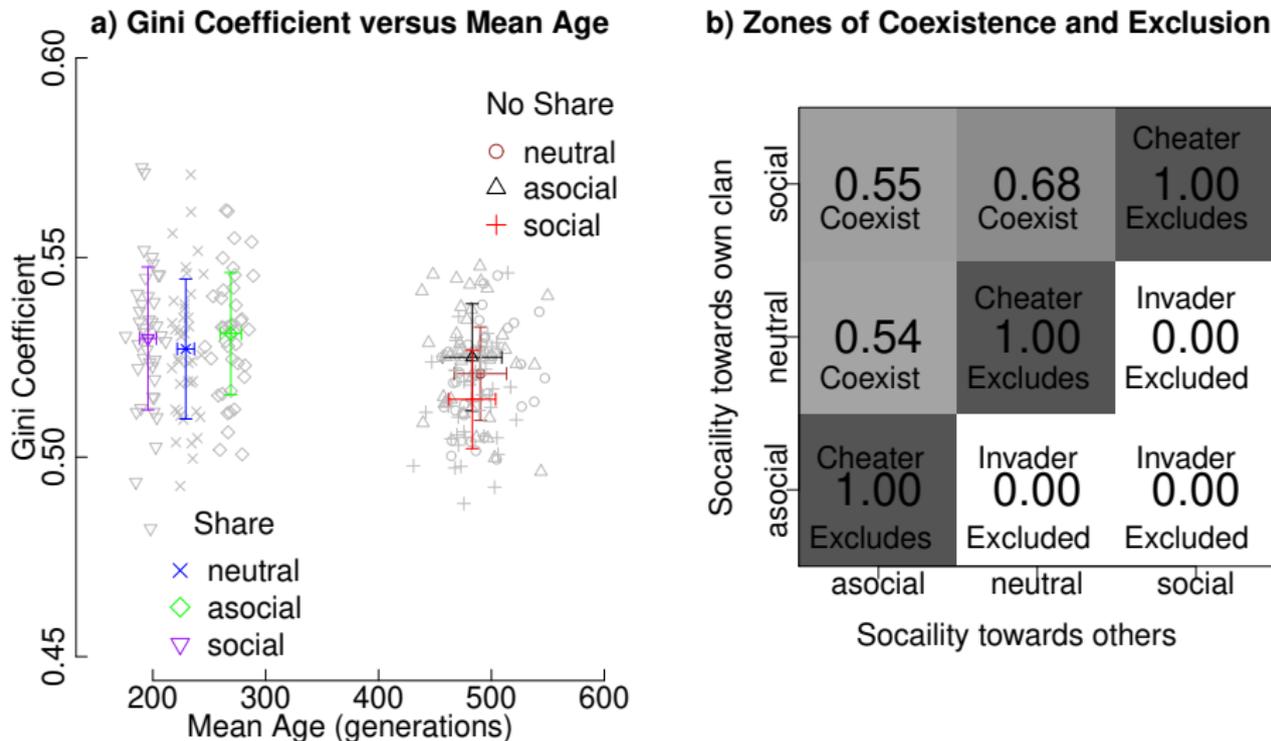


Figure: Gini Coefficient by mean age and zones of coexistence and exclusion

Coexistence of Different Sharing Clans

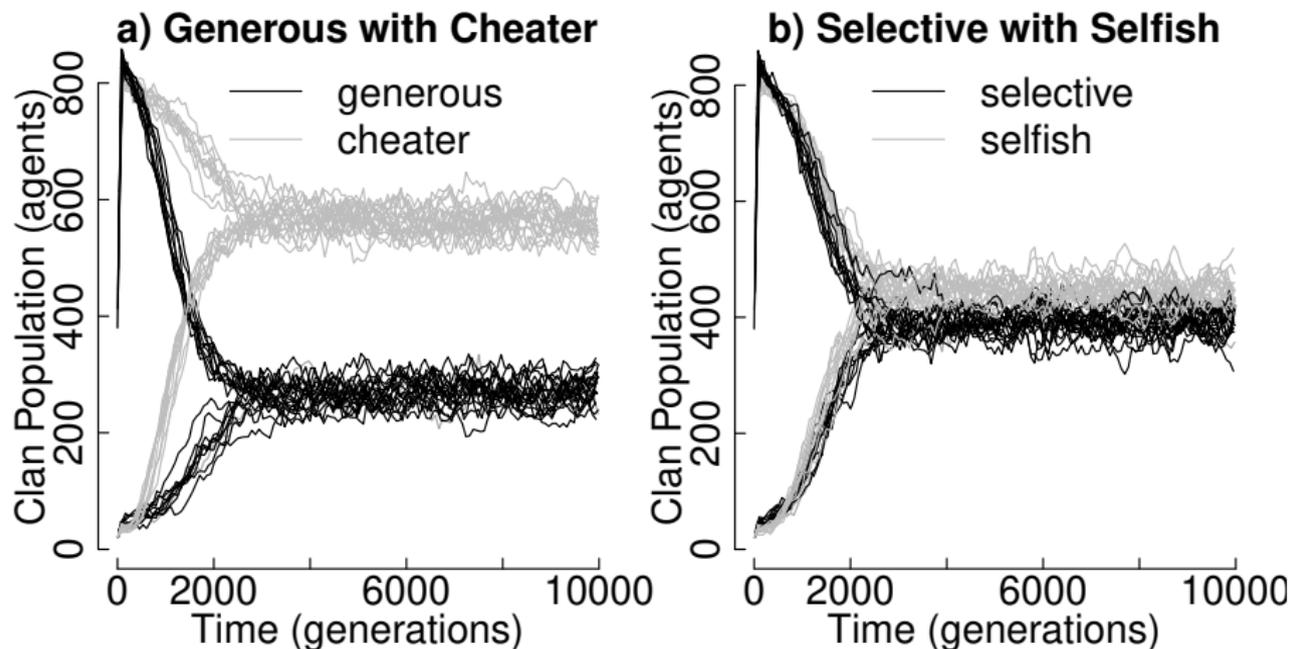


Figure: Coexistence of sharing alleles with selfish allele

Towards Eusociality

- Eusociality

- ▶ “one special difficulty” (Darwin, 1859)
- ▶ “one of the great mysteries of biology” (Howard and Thorne, 2011)
- ▶ Based on nesting bees (Batra, 1966)
- ▶ Bees, termites and ants (Wilson et al., 1971)
 - ★ Division of reproductive labor
 - ★ Cooperative care of the young
 - ★ Overlapping generations
 - ★ Phenotypic plasticity (implicit)
- ▶ Arthropods and vertebrates (Crespi and Yanega, 1995)
- ▶ Other common traits (Friedman et al., 2020)
 - ★ Nesting and defense of same
 - ★ Environmental effects on reproduction rate
 - ★ Coexistence of phenotypes
 - ★ Group foraging
 - ★ Haplodiploidy or similar reproduction

Inverse ABM

- Genome parameter space limited to **infertility and birth cost**
- Number of **instructions per action cycle** is third parameter searched
- (Queen's) **Genome held constant**, agent rules evolve

Name	Address	Function	Values	Description
nextI	1-2	register	05-32	address of next instruction
bDir	3	register	UDLRZ	best seen direction (Z=no data)
bDis	4	register	0-9	best seen distance
bRes	5	register	0-9	best seen resources
inst	6-32	program	UDLRMX	executeable instruction

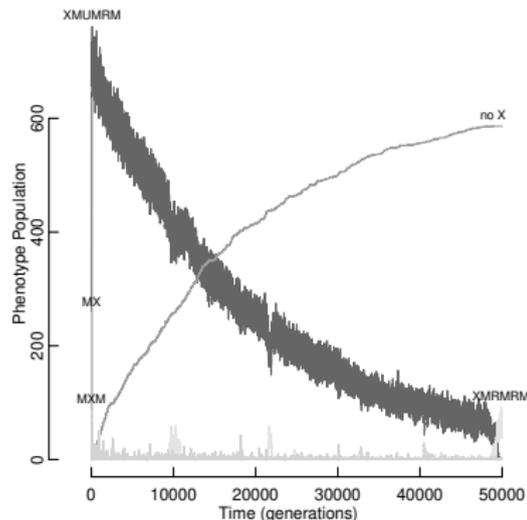
Instr	Description	Action/Test	Result
U	look up	find cell max resource above > bRes	store in bDir,bDis,bRes
D	look down	find cell max resource below > bRes	store in bDir,bDis,bRes
L	look left	find cell max resource left > bRes	store in bDir,bDis,bRes
R	look right	find cell max resource right > bRes	store in bDir,bDis,bRes
M	move	fetch bDis, bDir, if 'Z' random values	move bDis,bDir
X	reproduce	space, birth costs allow reproduction	place new agent in cell

Division of Reproductive Labor

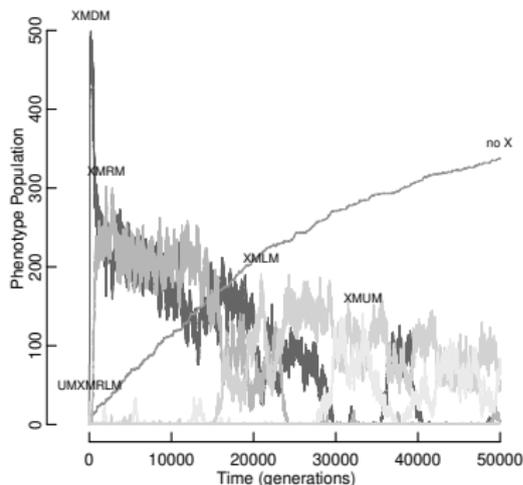
- no deaths by natural causes (no FL)
- CAS2
- losses ability to adapt to changing environment

Populations with Majority Non-reproducing Phenotypes

a) Birth Cost 1



b) Birth Cost 5



a) The emergence of a large population of non-reproducing phenotypes (no X) for infertility and birth cost 1, mutation rate 0.01, and computation capacity 2. b) The emergence of a large population of non-reproducing phenotypes (no X) for infertility 1 and birth cost 5, mutation rate 0.01, and computational capacity 2.

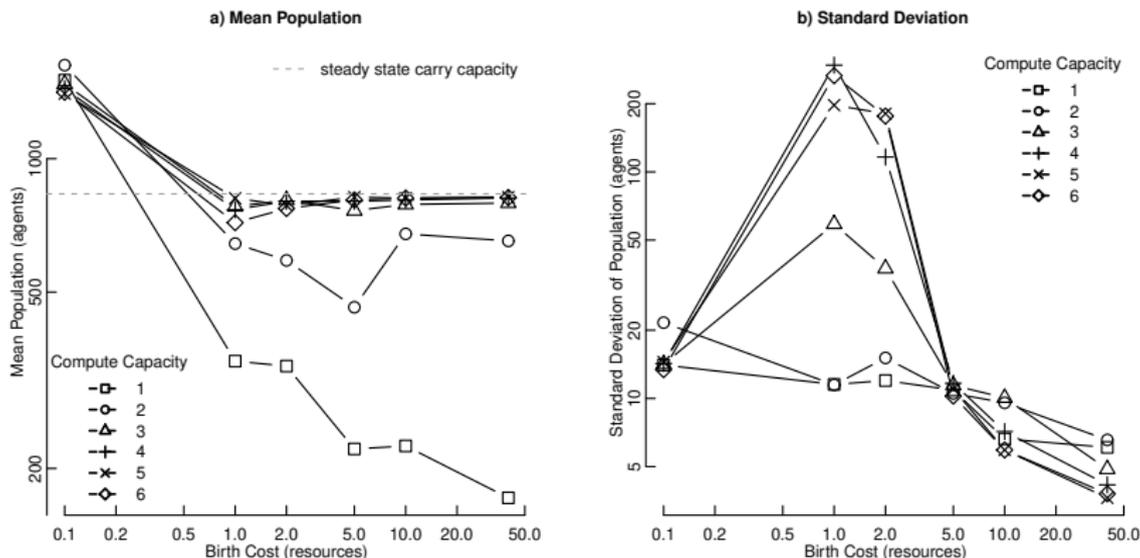
Novel Unexpected Solutions

- population exceeds the steady state carry capacity, K_T where

$$K_T = \frac{N_c}{\bar{m}} * g \quad (8)$$

- N_c number of landscape cells, g is resource growth per cell per cycle, \bar{m} is metabolism

Realized Carry Capacities and Volatilities



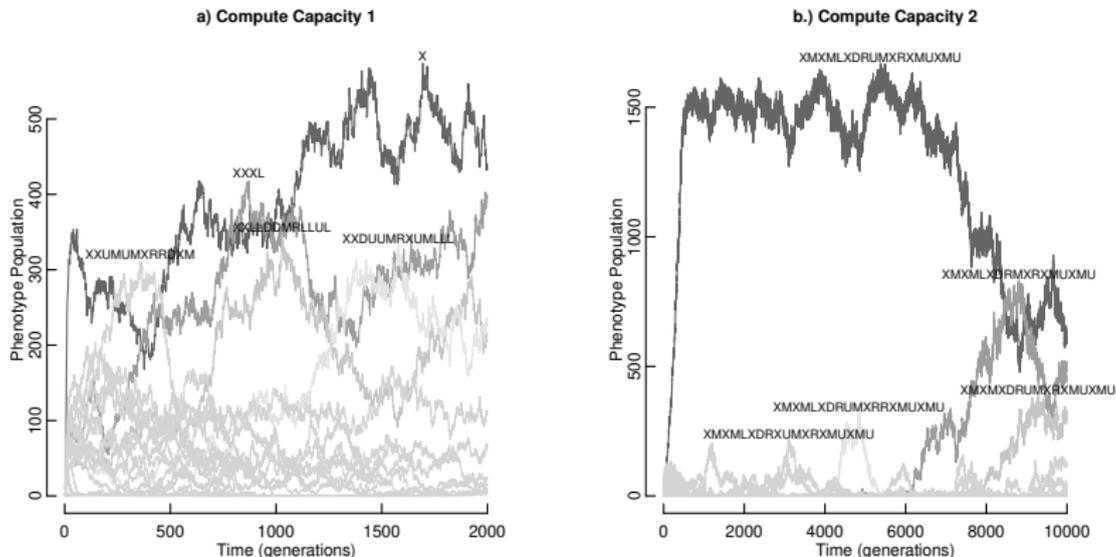
a) The steady state population levels as proxy for fitness versus birth cost across computational capacities for representative runs.

b) The volatility (standard deviation) of these steady state populations levels versus birth cost across computational capacities.

Phenotype Trajectories

- steady state carry capacity with non-stochastic infertility 1 and birth cost 0
- population level steady state, 92% of population dies each generation
- 95% of regrowth lost to foragers who immediately die

Phenotype Population Trajectories for Birth Cost Zero



Two representative examples with infertility 1 and birth cost 0. a) A successful phenotype of the single instruction X emerges with computational capacity 1. b) Foraging strategies emerge with computational capacity 2.

Applications - GP Bloat Hypothesis

- with birth cost 0 solutions, only first compute cycle instructions executed
- **Hypothesis:** longer genes due to bloat are protective against single gene mutation

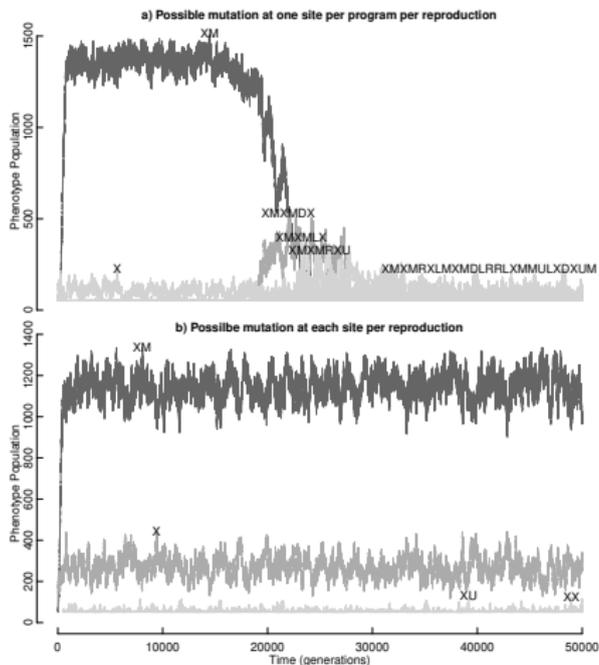


Figure: a) Bloat improves fitness when possible reproduction mutation is only once per program. b) Bloat provides no advantage when possible mutation is per instruction.

Death, Taxes and Inequality - Local Sharing

a. Mean Surplus versus Gini Coefficient

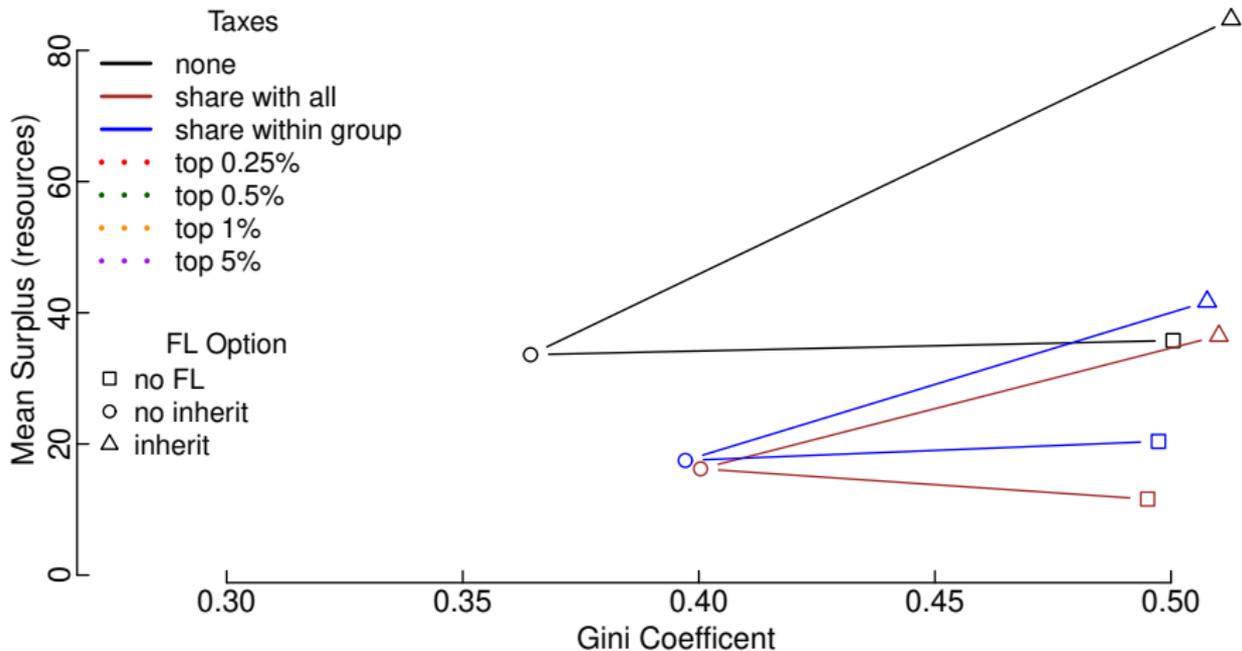


Figure: Simulations results, bottom up local sharing

Death, Taxes and Inequality - Taxes

a. Mean Surplus versus Gini Coefficient

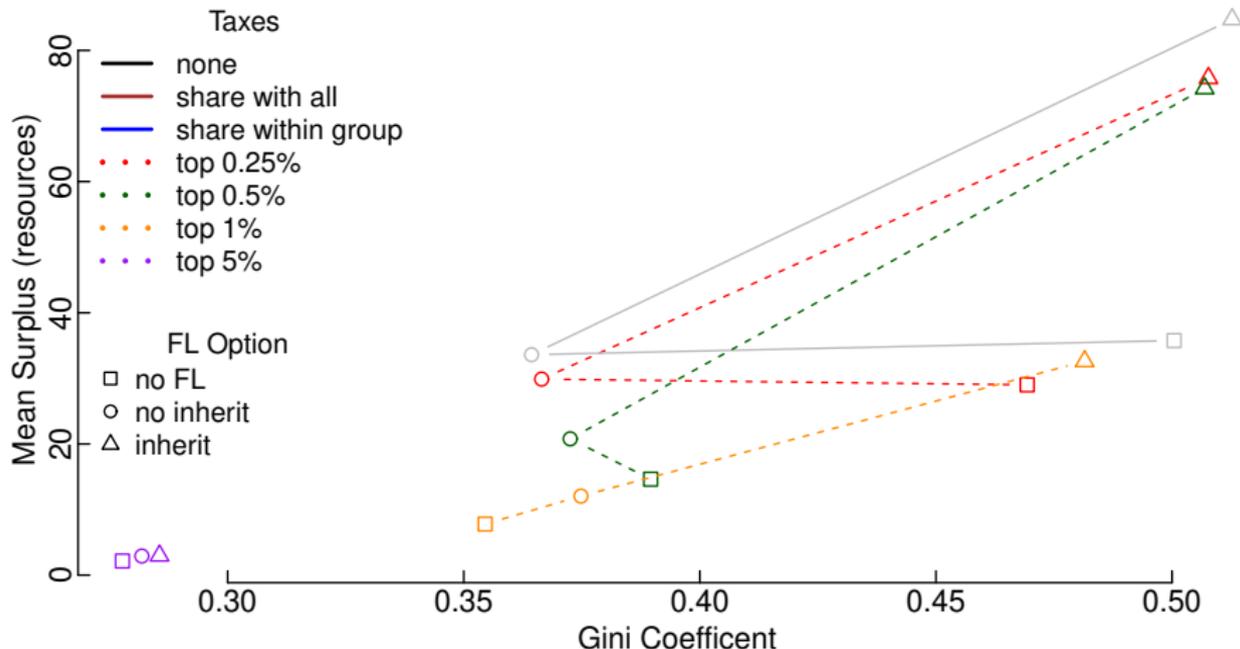


Figure: Simulations results, top down income and estate tax

Dangerous Allure of Low Fertility

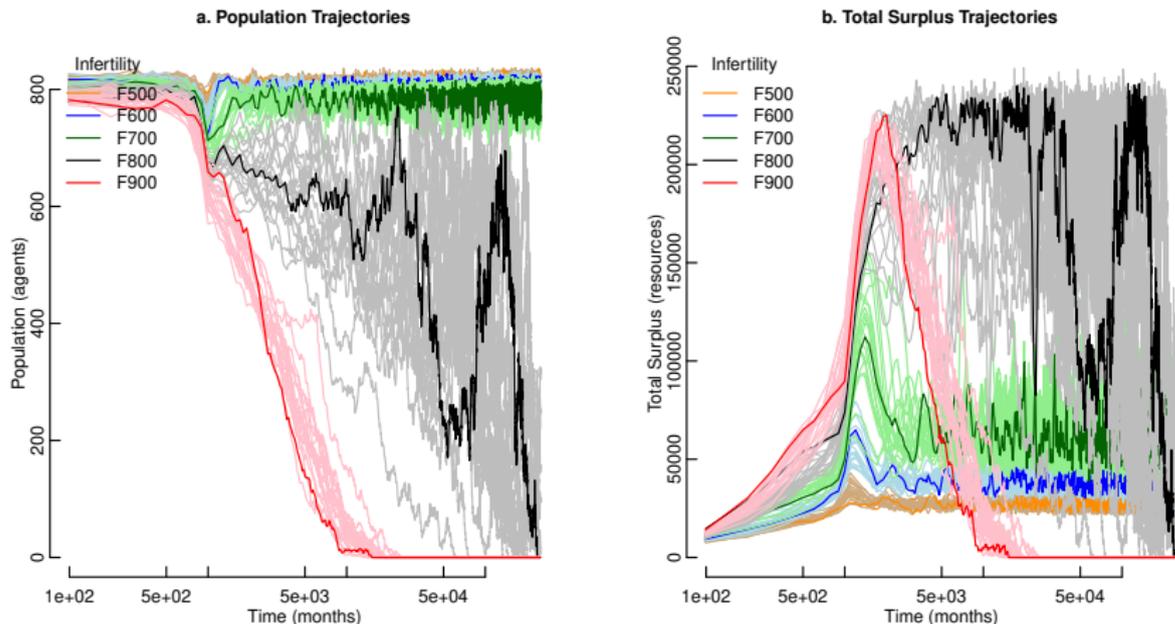
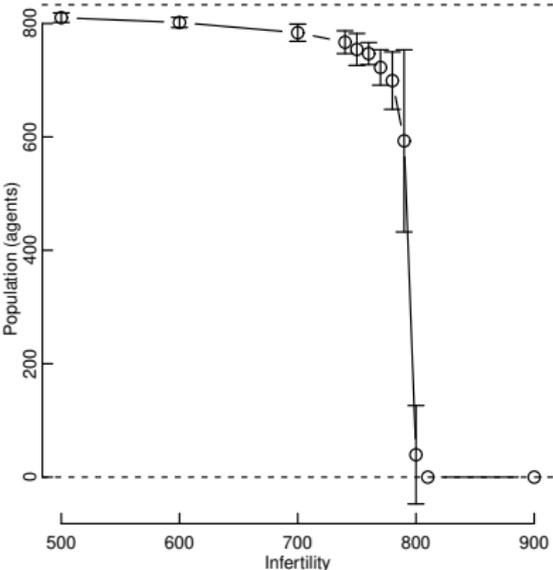


Figure: Dynamic population and total surplus trajectories for 20 differently seeded runs at high values of infertility. a) At some point (F800), the dynamics transition to a decay regime with extinction inevitable. b) At first a remarkable spike but stochastic extinction beckons.

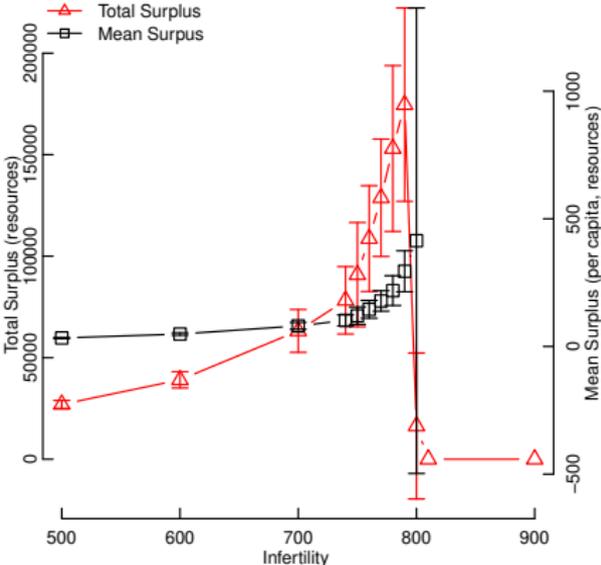
Dangerous Allure of Low Fertility (cont.)

These figures show the troubling increase in total surplus as the populations enter transition to the chaotic regime. As troubling, the mean surplus continues to increase as the population heads to extinction.

a. Steady State Population versus Infertility



b. Steady State Surpluses versus Infertility



Future Work

- Evolution of CAS1 societies with between group selection pressures
 - ▶ eusocial behaviors
 - ▶ fitness advantages of FL (death by natural causes)

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